Linear Algebra L3 - Linear mappings

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Learning Goals

- Understanding Linear mappings

Task 1

Draw these affine spaces (you can contribute to tree murdering via pen and paper, thats ok.)

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \cdot x + 1.5 = 0$$
$$\begin{bmatrix} -5 \\ -1 \end{bmatrix} \cdot x + 6 = 0$$
$$\begin{bmatrix} -5 \\ -1 \end{bmatrix} \cdot x - 6 = 0$$

Task 2

Find two non-parallel vectors x solving

$$w \cdot x = 3$$
$$w = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

Task 3

Show that the line given by

$$f(t) = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$$

does not intersect the plane given by

$$2x + z = 9$$

Note:

$$f(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

Task 4

Show that the line given by

$$f(t) = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$$

has an intersection with the plane given by

$$3x - 2y + 2z = 18$$

Note:

$$f(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

Task 5

Check whether the plane given by

$$f(s,t) = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

has an intersection with the line given by

$$x + 2y - z = 3$$
$$2x - y + z = 6$$

Note:

$$f(s,t) = \begin{bmatrix} x(s,t) \\ y(s,t) \\ z(s,t) \end{bmatrix}$$

Task 6

Convert the plane equation into the form $Ax = b$ for

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix} + s \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

Steps:

- what is the dimensionality of the whole vector space in which these equations are defined?
- what is the dimensionality of the affine space spanned by the plane equation?
- how does the matrix B look like for which we seek solutions x such that $Bx = 0$?
- Conclude based on the dimensionality of the whole vector space and the dimensionality of the plane, what is the dimensionality of solutions x which we are searching for ?
- find a basis for these solutions. Turn it into a matrix A
- get the correct bias vector b based the A which you found

Task 7

Convert the plane equation into the form $Ax = b$ for

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} -3 \\ 1 \\ 6 \end{bmatrix} + t \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}$$

Task 8

plot 2d planes in a 3d space using e.g. matplotlib.