

**INF 1004 Mathematics 2**  
**Tutorial #11**

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**Question 1**

- Compute the eigenvalues and eigenvectors for the matrices below.
- For one of these matrices compute the matrix  $P$  such that  $P^{-1}DP = A$ , and verify that  $PAP^{-1}$  is the diagonal matrix of the eigenvalues

$$A = \begin{bmatrix} 4 & \sqrt{15} \\ \sqrt{15} & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -4 \\ 4 & -6 \end{bmatrix}$$

**My Solution**

**Question 2**

- Compute an eigenvector for the eigenvalue  $x = 3$  for the below  $(3, 3)$ -matrix. Note: nobody asks you to compute its characteristic polynomial or to get all of its eigenvalues (Prof did it)
- Validate that the found eigenvector  $v$  is indeed the correct one, that is, that  $Av = 3v$  holds.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 2 & 4 & 4 \end{bmatrix}$$

**My Solution**

**Question 3**

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

- Show that this matrix has only the eigenvalue 1, twice.
- Prove that there cannot exist any matrix  $P$  such that  $P^{-1}DP = A$ . Hint: You know how  $D$  in  $P^{-1}DP$  must look like
- Find an eigenvector

Bonus knowledge: Shear matrices have an eigenspace of dimensionality  $d - 1$ . In the above case the set of all eigenvectors must be  $cv, c \in \mathbb{R}$  for some vector  $v$ .

**My Solution**

**Question 4**

Show that

$$\begin{bmatrix} 2 & -4 \\ \frac{13}{4} & -4 \end{bmatrix}$$

- has no real eigenvalue
- Bonus: what are its complex-valued eigenvalues?

**My Solution**

## Question 5

Bonus Matrix:

- Get its eigenvalues and eigenvectors

Note: This is a symmetric one, so you can expect 2 eigenvalues and orthogonal eigenspace

$$A = \begin{bmatrix} -2 & \sqrt{24} \\ \sqrt{24} & 8 \end{bmatrix}$$

**My Solution**

**Question 6**

- Use numpy to get its eigenvalues and eigenvectors
- Solve  $Ax = (3, 17, 1/3)$  using numpy
- Not in exam:

Compute the characteristic polynomial for

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & -2 & 1 \\ 3 & 1 & 3 \end{bmatrix}$$

**My Solution**

**Question 7**

What is the cosine of the angle between

$$(6, -6, -4, \sqrt{12}), (6, 4, 2, \sqrt{25})$$

**My Solution**

## Question 8

Another 3x3 affine system

- Show the intermediate result when the first column is the one hot vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  for the first time
- Show the intermediate result when the matrix has row echelon form for the first time
- Get the solution

$$\begin{aligned}2x - 3y + 2z &= -4 \\7x + 4.5y - 1z &= 16 \\4x + 3y + z &= 2\end{aligned}$$

**My Solution**