Linear Algebra L4 - Matrices

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1 Learning Goals

- matrix properties

Task 1

Compute $A^\top A$ for

$$A = \begin{bmatrix} 2 & -3 \\ -6 & -9 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$

Task 2

Compute the inverse of

$$A_0 = \begin{bmatrix} -2 & -3 \\ -6 & -4 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

Use these inverses to solve

$$A_0 x = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$A_1 x = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

Note: it is not common to solve $Ax = b$ using matrix inversion.

Reasons:

- $Ax = b$ can be solvable when A is not invertible
- It is often slower / more costly see e.g. https://gregorygundersen.com/blog/2020/12/09/matrix-inversion/

Task 3

Compute the determinant of

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -4 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 2 \\ -1 & 3 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -5 & -10 & -15 \\ 6 & 12 & 18 \end{bmatrix}$$

- Are they invertible?
- Which of them has full rank ? Which of them has lower rank and which one ?

Task 4

For what value a the matrix is not invertible ?

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 4 \\ -3 & 1 & 2 \end{bmatrix}$$

Task 5

Compute and apply the Householder matrix which makes transforms the first column of to a multiple of the first one-hot vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ for

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

and for (hint: here subtracting is nicer)

$$A = \begin{bmatrix} 1 & -4 & 3 \\ 3 & 1 & 1 \\ \sqrt{6} & 3 & 1 \end{bmatrix}$$

Task 6

Verify that

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

satisfies being an orthogonal matrix.

Task 7 (extra)

Because some of you had troubles with it
What is the cosine of the angle between

$$(6, -6, -4, \sqrt{12}),\ (6, 4, 2, \sqrt{25})\ ?$$

Task 8 (extra)

another 3x3 affine system

- show the intermediate result when the first column is the one hot vector $[1, 0, 0]$ for the first time
- show the intermediate result when the matrix has row echelon form for the first time
- get the solution

$$\begin{aligned} 2x - 3y + 2z &= -4 \\ 7x + 4.5y - 1z &= 16 \\ 4x + 3y + z &= 2 \end{aligned}$$