# INF 1004 Mathematics 2
# Tutorial #7

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# Question 1

Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 7 \end{pmatrix} \quad B = \begin{pmatrix} -2 & 5 \\ 0 & 8 \end{pmatrix} \quad C = \begin{pmatrix} 6 & 0 & 3 \\ 1 & 0 & -5 \end{pmatrix} \quad D = \begin{pmatrix} 4 & 0 & -4 \\ -3 & 4 & 9 \end{pmatrix}$$

Compute, if possible

(a) $A + B$
(b) $B + A$
(c) $A + B + C$
(d) $A + C$
(e) $(C^T)^T$
(f) $C + C^T$
(g) $5D - 3C$
(h) $5D^T - 3C^T$
(i) $(B^T - A^T)^T$

**Solution:**

(a) $\begin{pmatrix} 0 & 6 \\ 1 & 15 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 6 \\ 1 & 15 \end{pmatrix}$

(c) not possible

(d) not possible

(e) C

(f) not possible

(g) $\begin{pmatrix} 2 & 0 & -29 \\ -18 & 20 & 60 \end{pmatrix}$

(h) $\begin{pmatrix} 2 & -18 \\ 0 & 20 \\ -29 & 60 \end{pmatrix}$

(i) $\begin{pmatrix} -4 & 4 \\ -1 & 1 \end{pmatrix}$

## My Solution

### Part 1

$$A + B = \begin{pmatrix} 2 & 1 \\ 1 & 7 \end{pmatrix} + \begin{pmatrix} -2 & 5 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ 1 & 15 \end{pmatrix}$$

### Part 2

$$B + A = \begin{pmatrix} -2 & 5 \\ 0 & 8 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ 1 & 15 \end{pmatrix}$$

### Part 3

Not possible, since $A, B$ and $C$ have different dimensions.

### Part 4

Not possible, since $A$ and $C$ have different dimensions.

### Part 5

$$C^T = \begin{pmatrix} 6 & 1 \\ 0 & 0 \\ 3 & -5 \end{pmatrix}$$

$$(C^T)^T = C$$

### Part 6

Not possible, since $C$ and $C^T$ have different dimensions.

### Part 7

$$5D - 3C = 5\begin{pmatrix} 4 & 0 & -4 \\ -3 & 4 & 9 \end{pmatrix} - 3\begin{pmatrix} 6 & 0 & 3 \\ 1 & 0 & -5 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -29 \\ -18 & 20 & 60 \end{pmatrix}$$

Question 1 continued on next page...

## Part 8

$$\begin{aligned} 5D^T - 3C^T &= 5\begin{pmatrix} 4 & -3 \\ 0 & 4 \\ -4 & 9 \end{pmatrix} - 3\begin{pmatrix} 6 & 1 \\ 0 & 0 \\ 3 & -5 \end{pmatrix} \\ &= \begin{pmatrix} 20 & -15 \\ 0 & 20 \\ -20 & 45 \end{pmatrix} + \begin{pmatrix} -18 & -3 \\ 0 & 0 \\ 9 & 15 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -18 \\ 0 & 20 \\ -29 & 60 \end{pmatrix} \end{aligned}$$

## Part 9

$$\begin{aligned} (B^T - A^T)^T &= B - A \\ &= \begin{pmatrix} -2 & 5 \\ 0 & 8 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 7 \end{pmatrix} \\ &= \begin{pmatrix} -4 & 4 \\ -1 & 1 \end{pmatrix} \end{aligned}$$

# Question 2

Determine the constants $a, b, c$ and $d$, such that the following equation holds.

$$\begin{pmatrix} 0 & a & 3 \\ b & 2 & 2 \end{pmatrix} + \begin{pmatrix} c & 1 & 2 \\ 1 & 1 & d \end{pmatrix} = \begin{pmatrix} 4 & 3 & 5 \\ 7 & 3 & 5 \end{pmatrix}.$$

**Solution:** $a = 2, b = 6, c = 4, d = 3$

## My Solution

$$\begin{aligned} a + 1 &= 3 \\ a &= \boxed{2} \end{aligned}$$

$$\begin{aligned} b + 1 &= 7 \\ b &= \boxed{6} \end{aligned}$$

$$\begin{aligned} 0 + c &= 4 \\ c &= \boxed{4} \end{aligned}$$

$$\begin{aligned} 2 + d &= 5 \\ d &= \boxed{3} \end{aligned}$$

# Question 3

Given the matrices A and B, compute the products $AB$ and $BA$.

$$A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

**Solution:**

$$AB = \begin{pmatrix} -1 & 4 & 1 \\ 1 & -1 & 0 \\ 3 & 9 & 4 \end{pmatrix}, \quad BA = \begin{pmatrix} 5 & 1 \\ -1 & -3 \end{pmatrix}$$

## My Solution

### Part 1

$$\begin{aligned} AB &= \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 \cdot 1 + 2 \cdot (-1) & 1 \cdot 2 + 2 \cdot 1 & 1 \cdot 1 + 2 \cdot 0 \\ 0 \cdot 1 + (-1) \cdot (-1) & 0 \cdot 2 + (-1) \cdot 1 & 0 \cdot 1 + (-1) \cdot 0 \\ 4 \cdot 1 + 1 \cdot (-1) & 4 \cdot 2 + 1 \cdot 1 & 4 \cdot 1 + 1 \cdot 0 \end{pmatrix} \\ &= \boxed{\begin{pmatrix} -1 & 4 & 1 \\ 1 & -1 & 0 \\ 3 & 9 & 4 \end{pmatrix}} \end{aligned}$$

### Part 2

$$\begin{aligned} BA &= \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 4 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \cdot 1 + 2 \cdot 0 + 1 \cdot 4 & 1 \cdot 2 + 2 \cdot (-1) + 1 \cdot 1 \\ (-1) \cdot 1 + 1 \cdot 0 + 0 \cdot 4 & (-1) \cdot 2 + 1 \cdot (-1) + 0 \cdot 1 \end{pmatrix} \\ &= \boxed{\begin{pmatrix} 5 & 1 \\ -1 & -3 \end{pmatrix}} \end{aligned}$$

# Question 4

If A and B are invertible $n \times n$ matrices, use the definition of the inverse, to prove that

$$(AB)^{-1} = B^{-1}A^{-1}.$$

## My Solution

# Question 5

Solve for the matrix A,

$$\left[5A^T + \begin{pmatrix} 1 & 0 \\ 2 & 5 \end{pmatrix}\right]^T = 3A + \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}^T.$$

**Solution:**

$$A = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix}$$

## My Solution

$5\text{A} + \begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix} = 3\text{A} + \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$

$2\text{A} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ - $\begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix}$

$2\text{A} = \begin{pmatrix} 0 & -3 \\ -2 & -2 \end{pmatrix}$

$\text{A} = 0.5 \begin{pmatrix} 0 & -3 \\ -2 & -2 \end{pmatrix}$

# Question 6

In 2010, the average salary for all accountants in San Diego and Salt Lake City, was \$45, 091.50. The average salary in San Diego alone, however, was \$5231 greater than the average salary in Salt Lake City alone. What is the average salary of an accountant in each city, assuming that there are the same number of accountants in each city?

**Solution:** The average salary in San Diego and Salt Lake City is \$47,707 and \$42,476, respectively.

## My Solution

Let x: the average salary in San Diego
Let y: the average salary in Salt Lake City.

$$x + y = 45091.50 \times 2 \tag{1}$$

$$x - y = 5231 \tag{2}$$

Solving the equations simultaneously, we get:

$$\begin{aligned} x + y &= 90183 \\ x - y &= 5231 \\ x &= 47707 \\ y &= 42476 \end{aligned}$$

Therefore, the average salary of an accountant in San Diego is $\boxed{\$47,707}$ and in Salt Lake City is $\boxed{\$42,476}$.

# Question 7

A chemist has prepared two acid solutions, one of which is 2% by volume, the other 7%. How many cubic centimeters of each should the chemist mix together, to obtain $40cm^3$ of a 3.2% acid solution?

**Solution:** For the 2% solution is $30.4cm^3$ and for the 7% solution is $9.6cm^3$.

## My Solution

Let x: the volume of the 2% solution in $cm^3$
Let y: the volume of the 7% solution in $cm^3$.

$$\frac{2x + 7y}{40} = 3.2 \tag{3}$$

$$x + y = 40 \tag{4}$$

Solving the equations simultaneously, we get:

$$\begin{aligned} \frac{2x + 7y}{40} &= 3.2 \\ x + y &= 40 \\ x &= 30.4 \\ y &= 9.6 \end{aligned}$$

Therefore, the volume of the 2% solution is $\boxed{30.4\ \text{cm}^3}$ and the volume of the 7% solution is $\boxed{9.6\ \text{cm}^3}$.