INF 1004 Mathematics 2
Tutorial #7 Solution

Woon Jun Wei
2200624

February 25, 2023

Question 1

Compute the euclidean vector norm for vectors

$$\begin{bmatrix}1, 0, 2\end{bmatrix}, \begin{bmatrix}3, 4\end{bmatrix}, \begin{bmatrix}-7, 2, -4, \sqrt{12}\end{bmatrix}$$

My Solution

Part 1

$$\|[1, 0, 2]\|_2 = \sqrt{1^2 + 0^2 + 2^2} = \boxed{\sqrt{5}}$$

Part 2

$$\|[3, 4]\|_2 = \sqrt{3^2 + 4^2} = \sqrt{25} = \boxed{5}$$

Part 3

$$\begin{aligned}\|[-7, 2, -4, \sqrt{12}]\|_2 &= \sqrt{(-7)^2 + 2^2 + (-4)^2 + \sqrt{12}^2} \\ &= \sqrt{49 + 4 + 16 + 12} \\ &= \sqrt{81} \\ &= \boxed{9}\end{aligned}$$

Question 2

Compute the corresponding unit length vector for these:

$$[3,4]\,,\,[-1,-2,3]\,,\,[-7,2,-4,\sqrt{12}]$$

My Solution

$$\|v\|_2 = 1$$
$$v \neq 0 \implies \frac{v}{\|v\|_2}$$

Part 1

$$\begin{aligned}\frac{[3,4]}{\|[3,4]\|_2} &= \frac{[3,4]}{5}\\ &= \frac{1}{5}[3,4]\\ &= \boxed{\left[\tfrac{3}{5},\tfrac{4}{5}\right]}\end{aligned}$$

Part 2

$$\begin{aligned}\frac{[-1,-2,3]}{\|[-1,-2,3]\|_2} &= \frac{[-1,-2,3]}{\sqrt{1^2+4^2+9^2}}\\ &= \frac{1}{\sqrt{14}}[-1,-2,3]\\ &= \boxed{\left[-\tfrac{1}{\sqrt{14}},-\tfrac{2}{\sqrt{14}},\tfrac{3}{\sqrt{14}}\right]}\end{aligned}$$

Part 3

$$\begin{aligned}\frac{[-7,2,-4,\sqrt{12}]}{\|[-7,2,-4,\sqrt{12}]\|_2} &= \frac{[-7,2,-4,\sqrt{12}]}{\sqrt{(-7)^2+2^2+(-4)^2+\sqrt{12}^2}}\\ &= \frac{1}{\sqrt{81}}[-7,2,-4,\sqrt{12}]\\ &= \frac{1}{9}[-7,2,-4,\sqrt{12}]\\ &= \boxed{\left[-\tfrac{7}{9},\tfrac{2}{9},-\tfrac{4}{9},\tfrac{\sqrt{12}}{9}\right]}\end{aligned}$$

Question 3

Compute the inner product between these vectors and their angle in degrees:

$$\begin{bmatrix}3, -2, 2\end{bmatrix}, \begin{bmatrix}1, 2, 2\end{bmatrix}$$

Compute the inner product between these vectors and their angle in degrees:

$$\begin{bmatrix}1, 0, 1\end{bmatrix}, \begin{bmatrix}2, 1, -2\end{bmatrix}, \begin{bmatrix}\frac{1}{2\sqrt{2}}, -\frac{\sqrt{3}}{2}, \frac{1}{2\sqrt{2}}\end{bmatrix}$$

Sample Solutions

$$\frac{u}{\|u\|_2} \cdot \frac{v}{\|v\|_2} = \cos(\angle(u, v))$$

Part 1

$$\begin{aligned}\langle[3, -2, 2], [1, 2, 2]\rangle &= 3 \cdot 1 + (-2) \cdot 2 + 2 \cdot 2 \\ &= 3 + (-4) + 4 \\ &= \boxed{3}\end{aligned}$$

Angle:

Let u be $\begin{bmatrix}3, -2, 2\end{bmatrix}$ and v be $\begin{bmatrix}1, 2, 2\end{bmatrix}$

$$\begin{aligned}\cos(\angle(u, v)) &= \frac{\langle u, v\rangle}{\|u\|_2 \cdot \|v\|_2} \\ &= \frac{\begin{bmatrix}3, -2, 2\end{bmatrix} \cdot \begin{bmatrix}1, 2, 2\end{bmatrix}}{\|\begin{bmatrix}3, -2, 2\end{bmatrix}\|_2 \cdot \|\begin{bmatrix}1, 2, 2\end{bmatrix}\|_2} \\ &= \frac{3}{\sqrt{17} \cdot \sqrt{9}} \\ &= \frac{3}{\sqrt{153}} \\ &= \frac{1}{\sqrt{17}} \\ \angle(u, v) &= \cos^{-1}(\frac{3}{\sqrt{153}}) \\ &= \boxed{75.96^\circ} \textbf{ OR} \\ &= 360^\circ - 75.96^\circ \\ &= \boxed{284.04^\circ}\end{aligned}$$

Part 2

$$\begin{aligned}\langle[1,0,1],[2,1,-2]\rangle &= 1\cdot 2+0\cdot 1+1\cdot(-2)\\ &= 2+0+(-2)\\ &= \boxed{0}\end{aligned}$$

Angle:

Let u be $\begin{bmatrix}1,0,1\end{bmatrix}$ and v be $\begin{bmatrix}2,1,-2\end{bmatrix}$ and w be $\begin{bmatrix}\frac{1}{2\sqrt{2}},-\frac{\sqrt{3}}{2},\frac{1}{2\sqrt{2}}\end{bmatrix}$

$$\begin{aligned}\cos(\angle(u,v)) &= \frac{\langle u,v\rangle}{\|u\|_2\cdot\|v\|_2}\\ &= \frac{\begin{bmatrix}1,0,1\end{bmatrix}\cdot\begin{bmatrix}2,1,-2\end{bmatrix}}{\|\begin{bmatrix}1,0,1\end{bmatrix}\|_2\cdot\|\begin{bmatrix}2,1,-2\end{bmatrix}\|_2}\\ &= \frac{0}{\sqrt{2}\cdot\sqrt{6}}\\ &= 0\end{aligned}$$

$$\begin{aligned}\angle(u,v) &= \cos^{-1}(0)\\ &= \boxed{90^\circ}\ \textbf{OR}\\ &= 360^\circ-90^\circ\\ &= \boxed{270^\circ}\end{aligned}$$

Part 3

$$\left\langle[1,0,1]\cdot\left[\frac{1}{2\sqrt{2}},-\frac{\sqrt{3}}{2},\frac{1}{2\sqrt{2}}\right]\right\rangle = \frac{1}{\sqrt{2}}$$

$$\left\|\left[\frac{1}{2\sqrt{2}},-\frac{\sqrt{3}}{2},\frac{1}{2\sqrt{2}}\right]\right\| = \sqrt{\frac{1}{4\cdot 2}+\frac{3}{4}+\frac{1}{4\cdot 2}} = 1$$

$$\begin{aligned}\cos\angle(u,w) &= \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{1}\\ &= \frac{1}{2}\\ \angle(u,w) &= \cos^{-1}(\frac{1}{2})\\ &= 60^\circ\ \textbf{OR}\\ \angle(u,w) &= 360^\circ-60^\circ = 300^\circ\end{aligned}$$

Part 4

$$
\begin{aligned}
[2, 1, -2] \cdot \left[\frac{1}{2\sqrt{2}}, -\frac{\sqrt{3}}{2}, \frac{1}{2\sqrt{2}}\right] &= \frac{1}{2\sqrt{2}} \cdot 2 + \frac{\sqrt{3}}{2} \cdot 1 + \frac{1}{2\sqrt{2}} \cdot (-2) \\
&= -\frac{\sqrt{3}}{2}
\end{aligned}
$$

$$
\begin{aligned}
\cos \angle(v, w) &= -\frac{\sqrt{3}}{2} \cdot \frac{1}{3} \cdot \frac{1}{1} \\
&= -\frac{1}{2\sqrt{3}} \\
\angle(v, w) &= \cos^{-1}(-\frac{1}{2\sqrt{3}}) \\
&= 106.74^\circ \textbf{ OR} \\
\angle(v, w) &= 360^\circ - 106.74^\circ = 253.26^\circ
\end{aligned}
$$

Question 4

- What is the projection of $\left[5,2\right]$ onto the subspace spanned by vector $\left[1,1\right]$?
- What is the projection of $\left[0,2,1\right]$ onto the subspace spanned by vector $\left[1,-1,-1\right]$?
- Project $\left[5,2\right]$ onto the subspace spanned by vectors $\left[2,3\right]$, $\left[1,1\right]$
- What is the projection of $\left[1,-1,1\right]$ ~~onto the subspace spanned by vector $\left[1,1,1\right]$~~ onto the subspace spanned by vectors $\left[0,0,-1\right]$, $\left[2,0,1\right]$? Hint: this one is more tricky. Reason: $\left[0,-1,-1\right]\cdot\left[2,0,1\right]\neq 0$

My Solution

$$x_{\parallel}v = \frac{x\cdot v}{v\cdot v}v = \left(x\cdot\frac{v}{\|v\|_2}\right)\frac{v}{\|v\|_2}$$

Part 1

$$\begin{aligned}\frac{\left[5,2\right]\cdot\left[1,1\right]}{2}\left[1,1\right] &= \frac{7}{2}\left[1,1\right]\\ &= [3.5,3.5]\end{aligned}$$

Part 2

$$\begin{aligned}\frac{\left[0,2,1\right]\cdot\left[1,-1,-1\right]}{3}\left[1,-1,-1\right] &= \frac{-3}{3}\left[1,-1,-1\right]\\ &= \left[-1,1,1\right]\end{aligned}$$

Part 3

Use Gran Schmidts to get 2 orthogonal Vectors $\tilde{v}^{\{0\}}$ and $\tilde{v}^{\{1\}}$
Project $\left[5,2\right]$ onto the subspace spanned by $\tilde{v}^{\{0\}}$ and $\tilde{v}^{\{1\}}$ and sum up projections

$$\begin{aligned}\tilde{v}^{\{0\}} &= \left[1,1\right]\\ \tilde{v}^{\{1\}} &= \left[2,3\right]\\ \tilde{v}^{\{1\}} &= \left[2,3\right]-\frac{\left[2,3\right]\cdot\left[1,1\right]}{\left[1,1\right]\cdot\left[1,1\right]}\left[1,1\right]\\ &= \left[2,3\right]-\frac{5}{2}\left[1,1\right]\\ &= \left[-\tfrac{1}{2},\tfrac{1}{2}\right]\end{aligned}$$

Projection:

$$\begin{aligned}\frac{\left[5,2\right]\cdot\left[1,1\right]}{2}\left[1,1\right]+\frac{\left[5,2\right]\cdot\left[-\frac{1}{2},\frac{1}{2}\right]}{\left[-\frac{1}{2},-\frac{1}{2}\right]}\left[-\tfrac{1}{2},\tfrac{1}{2}\right] &= \frac{7}{2}\left[1,1\right]+\frac{-\frac{3}{2}}{\frac{1}{2}}\left[-\tfrac{1}{2},\tfrac{1}{2}\right]\\ &= \left[3.5,3.5\right]+\left[-\tfrac{3}{2},\tfrac{3}{2}\right]\\ &= \left[3.5-\tfrac{3}{2},3.5+\tfrac{3}{2}\right]\\ &= \left[5,2\right]\end{aligned}$$

It must be $\left[5,2\right]$.
Reason: $\left[2,3\right]\cdot\left[1,1\right]$ are 2 independent vectors $\mathbb{R}^2$ span the whole vector space.

Question 4 continued on next page. . .

2 independent vectors a,b, therefore dimension of subspace spanned by a and b is $\mathbb{R}^2$
$\begin{bmatrix}5,2\end{bmatrix}$ projected onto $\mathbb{R}^2$ is $\begin{bmatrix}5,2\end{bmatrix}$

Part 4

let $\begin{cases} v^{\{0\}} = \begin{bmatrix}0,0,-1\end{bmatrix} \\ \\ v^{\{1\}} = \begin{bmatrix}2,0,1\end{bmatrix} \end{cases}$

$$\begin{aligned} v^{\{1\}} &= \begin{bmatrix}2,0,1\end{bmatrix} - \frac{\begin{bmatrix}2,0,1\end{bmatrix} \cdot \begin{bmatrix}0,0,-1\end{bmatrix}}{\begin{bmatrix}0,0,-1\end{bmatrix} \cdot \begin{bmatrix}0,0,-1\end{bmatrix}} \begin{bmatrix}0,0,-1\end{bmatrix} \\ &= \begin{bmatrix}2,0,1\end{bmatrix} - \frac{1}{1} \begin{bmatrix}0,0,-1\end{bmatrix} \\ &= \begin{bmatrix}2,0,1\end{bmatrix} - \begin{bmatrix}0,0,-1\end{bmatrix} \\ &= \begin{bmatrix}2,0,0\end{bmatrix} \end{aligned}$$

Now project $\begin{bmatrix}1,-1,1\end{bmatrix}$ onto the subspace spanned by $v^{\{0\}}$ and $v^{\{1\}}$

$$\begin{aligned} x_{\parallel v} &= \frac{\begin{bmatrix}1,-1,1\end{bmatrix} \cdot \begin{bmatrix}0,0,-1\end{bmatrix}}{\begin{bmatrix}0,0,-1\end{bmatrix} \cdot \begin{bmatrix}0,0,-1\end{bmatrix}} \begin{bmatrix}0,0,-1\end{bmatrix} + \frac{\begin{bmatrix}1,-1,1\end{bmatrix} \cdot \begin{bmatrix}2,0,0\end{bmatrix}}{\begin{bmatrix}2,0,0\end{bmatrix} \cdot \begin{bmatrix}2,0,0\end{bmatrix}} \begin{bmatrix}2,0,0\end{bmatrix} \\ &= \frac{-1}{1} \begin{bmatrix}0,0,-1\end{bmatrix} + \frac{2}{4} \begin{bmatrix}2,0,0\end{bmatrix} \\ &= \begin{bmatrix}1,0,1\end{bmatrix} \end{aligned}$$

Question 5

Compute these matrix multiplications:

$$\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \end{bmatrix}$$

Question: Do you need more of them to practice? If so, you can do at home:

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 2.5 \\ -3.5 & 1.5 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

My Solution

Part 1

$$\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 2 \cdot (-1) + 1 \cdot (-4) & 2 \cdot 0 + 1 \cdot (-2) \\ 3 \cdot (-1) + (-2) \cdot (-4) & 3 \cdot 0 + (-2) \cdot (-2) \end{bmatrix} = \begin{bmatrix} -6 & -2 \\ 5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -3 \cdot 2 & -3 \cdot 4 & -3 \cdot (-2) \\ 2 \cdot 2 & 2 \cdot 4 & 2 \cdot (-2) \\ 1 \cdot 2 & 1 \cdot 4 & 1 \cdot (-2) \end{bmatrix} = \begin{bmatrix} -6 & -12 & 6 \\ 4 & 8 & -4 \\ 2 & 4 & -2 \end{bmatrix}$$

Part 2

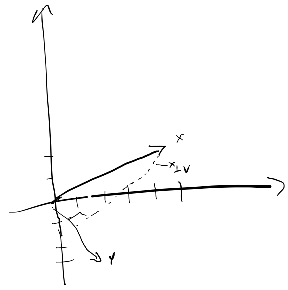
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 3 + 2 \cdot 0 & 1 \cdot 0 + 2 \cdot 1 & 1 \cdot 1 + 2 \cdot 2 \\ 2 \cdot 3 + 4 \cdot 0 & 2 \cdot 0 + 4 \cdot 1 & 2 \cdot 1 + 4 \cdot 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 6 & 4 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 2.5 \\ -3.5 & 1.5 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.5 \cdot 5 + 2.5 \cdot 4 \\ 0.5 \cdot 4 + 2.5 \cdot 6 \end{bmatrix} = \begin{bmatrix} 13 \\ -15 \end{bmatrix}$$

Question 6

- Project $\begin{bmatrix}5, 2\end{bmatrix}$ onto the orthogonal space of vector $\begin{bmatrix}2, -3\end{bmatrix}$
- Project $\begin{bmatrix}1, -1, 3\end{bmatrix}$ onto the orthogonal space of vector $\begin{bmatrix}-3, 1, 1\end{bmatrix}$
- Project $\begin{bmatrix}1, -1, 3, 1\end{bmatrix}$ onto the orthogonal space of vector $\begin{bmatrix}-2, 2, 0, 0\end{bmatrix}$, $\begin{bmatrix}0, 0, \sqrt{2}, \sqrt{2}\end{bmatrix}$

My Solution



Part 1

$$\begin{aligned}
x_{\perp v} &= \begin{bmatrix}5, 2\end{bmatrix} - \frac{\begin{bmatrix}5, 2\end{bmatrix} \cdot \begin{bmatrix}2, -3\end{bmatrix}}{\begin{bmatrix}2, -3\end{bmatrix} \cdot \begin{bmatrix}2, -3\end{bmatrix}} \begin{bmatrix}2, -3\end{bmatrix} \\
&= \begin{bmatrix}5, 2\end{bmatrix} - \frac{5 \cdot 2 + 2 \cdot (-3)}{2 \cdot 2 + (-3) \cdot (-3)} \begin{bmatrix}2, -3\end{bmatrix} \\
&= \begin{bmatrix}5, 2\end{bmatrix} - \frac{4}{13} \begin{bmatrix}2, -3\end{bmatrix} \\
&= \begin{bmatrix}5 - \frac{8}{13}, 2 + \frac{12}{13}\end{bmatrix} \\
&= \begin{bmatrix}\frac{57}{13}, \frac{38}{13}\end{bmatrix}
\end{aligned}$$

Verify:

$$\begin{bmatrix}5 - \frac{8}{13}, 2 + \frac{12}{13}\end{bmatrix} \cdot \begin{bmatrix}2, -3\end{bmatrix} = 0$$

Part 2

$$\begin{aligned}
x_{\perp v} &= \begin{bmatrix}1, -1, 3\end{bmatrix} - \frac{\begin{bmatrix}1, -1, 3\end{bmatrix} \cdot \begin{bmatrix}-3, 1, 1\end{bmatrix}}{\begin{bmatrix}-3, 1, 1\end{bmatrix} \cdot \begin{bmatrix}-3, 1, 1\end{bmatrix}} \begin{bmatrix}-3, 1, 1\end{bmatrix} \\
&= \begin{bmatrix}1, -1, 3\end{bmatrix} - \frac{1 \cdot (-3) + (-1) \cdot 1 + 3 \cdot 1}{(-3) \cdot (-3) + 1 \cdot 1 + 1 \cdot 1} \begin{bmatrix}-3, 1, 1\end{bmatrix} \\
&= \begin{bmatrix}1, -1, 3\end{bmatrix} - \frac{-1}{11} \begin{bmatrix}-3, 1, 1\end{bmatrix} \\
&= \begin{bmatrix}1 - \frac{3}{11}, -1 + \frac{1}{11}, 3 + \frac{1}{11}\end{bmatrix} \\
&= \begin{bmatrix}\dfrac{8}{11}, -\dfrac{10}{11}, \dfrac{34}{11}\end{bmatrix}
\end{aligned}$$

Verify:

$$\begin{bmatrix}1 - \frac{3}{11}, -1 + \frac{1}{11}, 3 + \frac{1}{11}\end{bmatrix} \cdot \begin{bmatrix}-3, 1, 1\end{bmatrix} = 0$$

Question 6 continued on next page...

Part 3

$$\left[-2, 2, 0, 0\right] \cdot \left[0, 0, \sqrt{2}, \sqrt{2}\right] = 0, \text{ thus:}$$

$$\begin{aligned}
x_{\perp v} &= \left[1, -1, 3, 1\right] - \frac{\left[1, -1, 3, 1\right] \cdot \left[-2, 2, 0, 0\right]}{\left[-2, 2, 0, 0\right] \cdot \left[-2, 2, 0, 0\right]} \left[-2, 2, 0, 0\right] - \frac{\left[1, -1, 3, 1\right] \cdot \left[0, 0, \sqrt{2}, \sqrt{2}\right]}{\left[0, 0, \sqrt{2}, \sqrt{2}\right] \cdot \left[0, 0, \sqrt{2}, \sqrt{2}\right]} \left[0, 0, \sqrt{2}, \sqrt{2}\right] \\
&= \left[1, -1, 3, 1\right] - \frac{-4}{8} \left[-2, 2, 0, 0\right] - \frac{0}{2} \left[0, 0, \sqrt{2}, \sqrt{2}\right] \\
&= \left[1, -1, 3, 1\right] - \left[-1, 1, 0, 0\right] - \left[0, 0, 2, 2\right] \\
&= \left[0, 0, 1, -1\right]
\end{aligned}$$

Verify:
Verify that $\left[0, 0, 1, -1\right] \cdot \left[-2, 2, 0, 0\right] = 0$ and $\left[0, 0, 1, -1\right] \cdot \left[0, 0, \sqrt{2}, \sqrt{2}\right] = 0$.

Question 7

Run Gram-Schmid-orthogonalization on the vectors

$$\begin{bmatrix}12, 12, 6\end{bmatrix}, \begin{bmatrix}2, -2, 4\end{bmatrix}, \begin{bmatrix}-2, -2, 1\end{bmatrix}$$

My Solution

Part 1

$$\tilde{v}^{\{0\}} = \begin{bmatrix}12, 12, 6\end{bmatrix}$$

$$\begin{aligned}
\tilde{v}^{\{1\}} &= \begin{bmatrix}2, -2, 4\end{bmatrix} - \frac{\begin{bmatrix}2, -2, 4\end{bmatrix} \cdot \begin{bmatrix}12, 12, 6\end{bmatrix}}{\begin{bmatrix}12, 12, 6\end{bmatrix} \cdot \begin{bmatrix}12, 12, 6\end{bmatrix}} \begin{bmatrix}12, 12, 6\end{bmatrix} \\
&= \begin{bmatrix}2, -2, 4\end{bmatrix} - \frac{2 \cdot 12 + (-2) \cdot 12 + 4 \cdot 6}{12 \cdot 12 + 12 \cdot 12 + 6 \cdot 6} \begin{bmatrix}12, 12, 6\end{bmatrix} \\
&= \begin{bmatrix}2, -2, 4\end{bmatrix} - \frac{24}{288 + 36} \begin{bmatrix}12, 12, 6\end{bmatrix} \\
&= \begin{bmatrix}2, -2, 4\end{bmatrix} - \left[\frac{12 \cdot 24}{9 \cdot 36}, \frac{12 \cdot 24}{9 \cdot 36}, \frac{6 \cdot 24}{9 \cdot 36}\right] \\
&= \begin{bmatrix}2, -2, 4\end{bmatrix} - \left[\frac{288}{324}, \frac{288}{324}, \frac{144}{324}\right] \\
&= \left[2 - \frac{8}{9}, -2 - \frac{8}{9}, 4 - \frac{4}{9}\right]
\end{aligned}$$

$$\begin{aligned}
\tilde{v}^{\{2\}} &= \begin{bmatrix}-2, -2, 1\end{bmatrix} \\
&- \frac{\begin{bmatrix}-2, -2, 1\end{bmatrix} \cdot \begin{bmatrix}12, 12, 6\end{bmatrix}}{\begin{bmatrix}12, 12, 6\end{bmatrix} \cdot \begin{bmatrix}12, 12, 6\end{bmatrix}} \begin{bmatrix}12, 12, 6\end{bmatrix} \\
&- \frac{\begin{bmatrix}-2, -2, 1\end{bmatrix} \cdot \left[2 - \frac{8}{9}, -2 - \frac{8}{9}, 4 - \frac{4}{9}\right]}{\left[2 - \frac{8}{9}, -2 - \frac{8}{9}, 4 - \frac{4}{9}\right] \cdot \left[2 - \frac{8}{9}, -2 - \frac{8}{9}, 4 - \frac{4}{9}\right]} \left[2 - \frac{8}{9}, -2 - \frac{8}{9}, 4 - \frac{4}{9}\right] \\
&= \begin{bmatrix}-2, -2, 1\end{bmatrix} - \frac{(-2) \cdot 12 + (-2) \cdot 12 + 1 \cdot 6}{12 \cdot 12 + 12 \cdot 12 + 6 \cdot 6} \begin{bmatrix}12, 12, 6\end{bmatrix} \\
&- \frac{(-2) \cdot 2 + (-2) \cdot (-2) + 1 \cdot 4}{2 \cdot 2 + (-2) \cdot (-2) + 4 \cdot 4} \left[2 - \frac{8}{9}, -2 - \frac{8}{9}, 4 - \frac{4}{9}\right] \\
&= \begin{bmatrix}-2, -2, 1\end{bmatrix} - \left[\frac{14}{9}, \frac{14}{9}, \frac{7}{9}\right] - \frac{\frac{36+28}{9}}{24 + \frac{128}{28} - \frac{32 \cdot 9}{81}} \left[2 - \frac{8}{9}, -2 - \frac{8}{9}, 4 - \frac{4}{9}\right]
\end{aligned}$$

Question 8

Understanding Distances coming from ℓ_p-norms

Coding: plot in python or similar the set of points $x \in \mathbb{R}^2$ usuch that $\|x\|_p = 1$ for

- $p = 0.2$
- $p = 0.5$
- $p = 1$
- $p = 1.5$
- $p = 2$
- $p = 4$
- $p = 8$
- $p = 16$

Hint: in 2 dimensions for $p = 2$ the solution is given by

$$x(t) = (\cos(t), \sin(t))$$

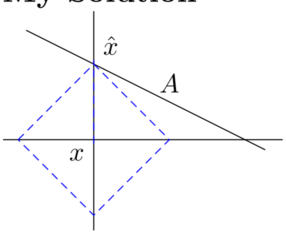
due to $\cos^2(t) + \sin^2(t) = 1$.
you can use the same idea with different powers. You can start by considering $(\cos^r(t), \sin^r(t))$. One thing to node: $\cos(t)^r + \sin(t)^r$ is not always defined for negative values and certain r.

For $p \neq 2$, you can consider this, thich deals with the signs:

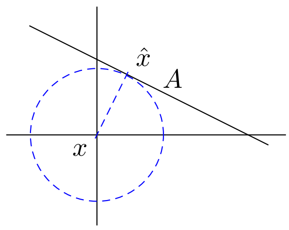
$$x(t) = (sign(\cos(t))|\cos(t)|^r, sign(\sin(t))|\sin(t)|^r)$$

for the right choice of r. Find out which r is suitable for a general $p > 0$ such that $\|x\|_p = 1$. Then plot it in python.

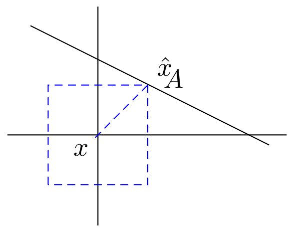
My Solution



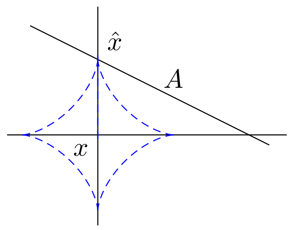
$p = 1$



$p = 2$



$p = \infty$



$p = \frac{1}{2}$