

MATH2: Week 4-5 Tutorial

Q1. Suppose that buses arrive at a bus-stop at every 10-min interval. What is the probability that a passenger who arrives at the bus-stop will have to wait between 5-7 mins. Assume all possibilities are equally likely.

Hints:

- Is this a uniform distribution or a normal distribution?
- What is the minimum in this distribution?
- What is the maximum in this distribution?
- Draw the probability distribution for this problem.

Q2. Stanford-Binet IQ Test scores are normally distributed with a mean score of 100 and a standard deviation of 16. Find the probability that a randomly selected person has an IQ test score of:

- (a) Over 140;
- (b) Under 88;
- (c) Between 72 and 128.

Q3. An advertising agency conducted an ad campaign aimed at making consumers aware of a new product. Upon completion of the campaign, the agency claimed that 20% of consumers had become aware of the product (fixed 20% chance of a customer being aware of the product). Assume that each customer's chance of being aware of the product is independent. The product's distributor surveyed 1,000 consumers and found that 156 were aware of the product. Assume we believe the claim of the agency.

- (a) Let X be the number of consumers who are aware of the product. What is the suitable probability distribution model for X ?
- (b) Calculate the mean μ and standard deviation σ of X .
- (c) Find the probability that 156 or fewer consumers in a random sample of 1,000 would be aware of the product.

Q4. The time, X minutes, taken by Nicholas to install a satellite dish may be assumed to be a normal random variable with mean 134 and standard deviation 16.

- (a) Determine the $P(X < 150)$.
- (b) Determine, the time exceeded by 10% of installations (only 10% of the installation times exceed this value).
- (c) The time taken, Y minutes, taken by Nicholas to install a satellite dish may also be assumed to be a normal random variable but with:

$$P(Y < 170) = 0.14 \text{ and } P(Y > 200) = 0.03$$

Determine the values for the mean and standard deviation of Y .

Q5. A strange new metal is discovered, and some coins are made from it. You're unsure if the properties of the metal will affect the fairness of the coins. However, based on some initial coin flips you think that 65% of the time the coins land on Tails, while 35% of the time it's Heads. Each coin flip is independent from the others and the probabilities are fixed. Assuming that we want to get as many Tails as possible for 1000 coin flips, answer the following:

- a. What type of probability distribution is suitable for this model?
- b. What is the mean and standard deviation for the 1000 coin flips?
- c. After the 1000 coin flips, 599 Tails are counted. Based on the current distribution, how likely is there to be 599 Tails or less?

Q6. Based on past data, an insurance company determines that drivers in the age group of 21–28 have an 80% chance of no accidents in a year, a 20% chance of being in a single accident, and no chance of being in more than one accident in a year. For simplicity, assume that after an accident, there is a 50% probability that the car will need repairs costing \$500, a 40% probability of repairs costing \$5,000, and a 10% probability of repairs costing \$15,000.

- (a) What is the expected repair cost?
- (b) What is the potential variability of the repair cost, i.e. variance and standard deviation?
- (c) Assume the insurer sells the policy to 100 car owners, each with the same risk, what is the insurer's total risk?
- (d) To help cover the variability and other expenses, the insurer decided to charge an additional 30% over the expected repair cost. What would be the gross premium for a policy?

Sampling Distribution Questions

Q7. A wholesaler sells a large number of 2-litre and 5-litre tubs of ice-cream which are sold in the ratio 3:2. A random sample of 2 tubs, (X_1, X_2) is taken from the shelves.

- (a) Find the mean and variance of the ice-cream content in this population.
- (b) List all the possible samples.
- (c) Find the sampling distribution for the mean \bar{X} .

Q8. A supermarket sells a large number of packets of bolts. One packet contains 10 bolts and the other 20 bolts. They are sold in the ratio 1:4 respectively. A random sample of 3 packets (X_1, X_2, X_3) is taken from the shelves.

- (a) List all the possible samples.
- (b) Find the sampling distribution for the mode N and the median M .

Q9. A random sample of 100 observations is to be drawn from a population with a mean of 40 and a standard deviation of 25. The population is a right-skewed distribution.

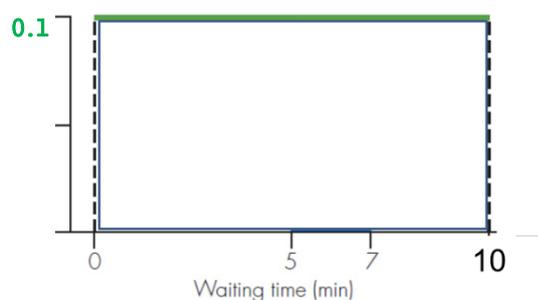
- Give the mean and standard deviation of the sampling distribution of mean of \bar{x} .
- Will the sampling distribution of mean of \bar{x} be approximately normal? Explain.
- Find the approximate probability that the mean of the sample will exceed 45.

Q10. In an article in the Journal of American Pediatric Health researchers claim that the weights of healthy babies born in the United States form a distribution that is nearly Normal with an average weight of 7.25 pounds and standard deviation of 1.75 pounds. Suppose a researcher selects 50 random samples with 30 newborns in each sample.

- What is the best estimate for the mean of the sample means?
- What is the best estimate of the standard deviation of the sample means?
- If we randomly selected 30 newborns from the full population of US newborns, would you be surprised if their mean weight was 8.30 pounds?

Solutions:

Q1. Uniform distribution, Min = 0, Max = 6



Ans = 0.2

Q2.

- 0.0062
- 0.2266
- 0.9198

Q3.

- Binomial distribution
- Mean = 200, Stdev = 12.65
- 0.0003

Q4.

- a) 0.8413
- b) 154.48
- c) Mean = 180.9462, Stdev = ~10.135

Q5.

- a) Binomial Random variable $X \sim B(n, p)$
 $B(1000, 0.65)$
- b) $\mu = np, \sigma^2 = npq$
 $\mu = 650, \sigma = \sqrt{227.5} = 15.0831$
- c) $P(Z \leq -3.3481) = 1 - 0.9996$
 $= 0.0004$

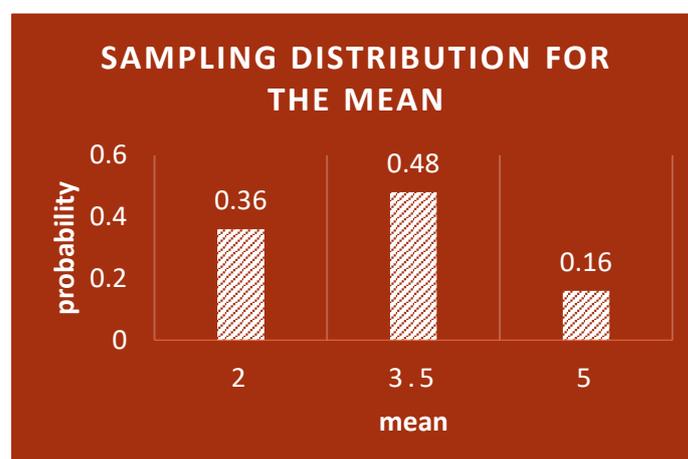
Q6.

- a) \$750
- b) Variance = \$5,962,500, Stdev = \$2442
- c) CV = 0.326
- d) $1.3 * E(X) = \$975$

Q7.

- a) Mean: 3.2 L, Variance = 2.16 L
- b) (2,2), (5,2), (2,5), (5,5)
- c)

Sample	\bar{x}	$P(\bar{x})$
(2,2)	2	$\left(\frac{3}{5}\right)^2 = \frac{9}{25}$
(5,2), (2,5)	3.5	$\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)\left(\frac{3}{5}\right) = \frac{12}{25}$
(5,5)	5	$\left(\frac{2}{5}\right)^2 = \frac{4}{25}$



Q7.

a)

b)

Sample	mode	P(mode)	median	P(median)
(10,10,10)	10	$\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$	10	Same as mode
(10,20,10) (20,10,10)(10,10,20)	10	$\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right) 3$	10	
(10,20,20) (20,20,10)(20,10,20)	20	$\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) 3$	20	
(20,20,20)	20	$\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)$	20	

Q8.

a) Mean = 40

Sampling Stdev = 2.5

b) Yes, normally because $n > 30$.

c) 0.0228

Q9.

a) 7.25

b) 0.32

c) Yes, a mean of 8.30 pounds would be surprising as this sample result is more than 3 standard deviations above the overall mean weight of 7.25.