INF 1004 Mathematics 2
Probability Practice Exercises

Woon Jun Wei
2200624

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Question 1

A jar contains four coins: a nickel (5 cents), a dime (10 cents), a quarter (25 cents), and a half-dollar (50 cents). Three coins are randomly selected without replacement from the jar.

(a) List all the possible outcomes in sample space S.

(b) What is the probabilty that the selection will contain the half-dollar?

(c) What is the probability that the total amount drawn will equal 65 cents or less?

(d) Which interpretations of probability are you applying?

My Solution

Part 1

$$S = \{(5, 10, 25), (5, 25, 50), (10, 25, 50), (5, 10, 50)\}$$

Part 2

$$P(\text{half dollar}) = \frac{3}{4}$$

Part 3

Sum of the outcomes:

$$S = \{40, 80, 85, 65\}$$

$$P(\text{total} \leq 65) = \frac{2}{4} = \frac{1}{2}$$

Part 4

Classical interpretation.

Question 2

In a genetics experiment, the researcher mated two Drosophila fruit flies and observed the traits of 300 offsprint. The results are shown in the table

	Wing Size	
Eye Color	Normal	Miniature
Normal	140	6
Vermillion	3	151

One of these offspring is randomly selected and observed for the two genetic traits

(a) What is the probability that the fly has normal eye color and normal wing size?

(b) What is the probability that the fly has vermillion eyes?

(c) What is the probability that the fly has either vermillion eyes or miniature wings, or both?

(d) Which interpretations of probability are you applying?

My Solution

Part 1

$$P(\text{Normal Eye Color} \cap \text{Normal Wing Size}) = \frac{140}{300} = \frac{7}{15} = 0.46667$$

Part 2

$$P(\text{Vermillion Eyes}) = \frac{3+151}{300} = \frac{77}{150} = 0.513$$

Part 3

$$\begin{aligned} P(\text{vermillion eyes } \cup \text{ miniature wings}) &= P(\text{vermillion eyes}) + P(\text{miniature wings}) - P(\text{vermillion eyes} \cap \text{miniature wing} \\ &= \frac{154}{300} + \frac{157}{300} - \frac{151}{300} \end{aligned}$$

Part 4

Relative frequency interpretation.

Question 3

City residents were surgeyed recently to determine readership of newspapers. 50% of the residents read the morning paper, 60% read the evening paper and 20% read both newspapers. Find the probability that a resident selected at random reads the morning or evening paper

My Solution

Given:
$P(\text{morning}) = 0.5$
$P(\text{evening}) = 0.6$
$P(\text{both}) = 0.2 = P(\text{morning } \cap \text{ evening})$

To find: $P(\text{morning or evening})$

$$\begin{aligned} P(\text{morning} \cup \text{evening}) &= P(\text{morning}) + P(\text{evening}) - P(\text{morning} \cap \text{evening}) \\ &= 0.5 + 0.6 - 0.2 \\ &= 0.9 \end{aligned}$$

Question 4

The experience of a train passenger is that the train is cancelled with probability 0.02 When it does run, it has probability of 0.9 of being on time.

(a) Find the probability that the train runs on time

(b) Given that the train is not on time, find the probability that it has been cancelled

My Solution

Given:
P(cancelled) = 0.02
P(cancelled^c) = 0.98
$P(\text{on time}|\text{cancelled}^c) = 0.9$

Part 1

$$P(\text{on time}) = 0.98 \cdot 0.9 = 0.882$$

Part 2

From part (a):

$$P(\text{on time}^c) = 1 - 0.882 = 0.118$$

$$\begin{aligned} P(\text{cancelled}|\text{on time}^c) &= \frac{P(\text{cancelled} \cap \text{on time}^c)}{P(\text{on time}^c)} \\ &= \frac{0.02 \cdot 1}{0.02 \cdot 1 + 0.98 \cdot 0.1} \\ &= 0.1694915 \\ &\approx 0.1694 \end{aligned}$$

Draw the probability tree diagram to visualise.
Note: Once the train is cancelled, there is **NO WAY** the train can be on time,
Therefore, $P(\text{on time}|\text{cancelled}) = 0$ and $P(\text{on time}^c|\text{cancelled}) = 1$.

Question 5

In a group of students 60% are female and 40% are male. A third of the female students study french but only a quarter of the male students study french
A student is chosen at random from the group.

(a) Show that the probability that the chosen student is female and studies french is 0.2

(b) Calculate the probability that the chosen student studies french

(c) Given that the chosen student does study french, calculate the conditional probability that the chosen student is female

My Solution

Let f: female
fr: studies french

Given: $P(f) = 0.6$
$P(f^c) = 0.4$
$P(fr) = 0.3$
$P(fr^c) = 0.7$
$P(fr|f) = 0.3$
$P(fr^c|f) = 0.7$
$P(fr|f^c) = 0.25$
$P(fr^c|f^c) = 0.75$

Draw the probability tree to help solve the problem

Part 1

$$P(f \cap fr) = P(f) \cdot P(fr|f) = 0.6 \cdot \frac{1}{3} = 0.2$$

Part 2

$$P(fr) = 0.2 + (0.4 \cdot 0.25) = 0.2 + 0.1 = 0.3$$

Part 3

$$P(f|fr) = \frac{P(f \cap fr)}{P(fr)} = \frac{0.2}{0.3}$$

Question 6

Of the patients in an emergency unit, 30% have a sports injury. Pass records for this emergency unit suggest that :

A patient with sports injury has a probability of 0.2 of being admitted to hospital

A patient who doesn't have a sports injury has a probability of 0.4 of being admitted to hospital

Now, a patient is chosen at random from those in the emergency unit.

(a) What is the probability that the chosen patient has a sports injury

(b) What is the probability that the chosen patient has a sports injury and is admitted to hospital

(c) Show that the probability that the chosen patient is admitted to hospital is 0.34

(d) Given that the chosen patient is admitted to hospital, find the conditional probability that the patient has a sports injury

My Solution

Given:
$P(\text{sports injury}) = 0.3$
$P(\text{sports injury}^c) = 0.7$
$P(\text{admitted}|\text{sports injury}) = 0.2$
$P(\text{admitted}|\text{sports injury}^c) = 0.4$

Part 1

$$P(\text{sports injury}) = 0.3$$

Part 2

$$P(\text{sports injury} \cap \text{admitted}) = P(\text{sports injury}) \cdot P(\text{admitted}|\text{sports injury}) = 0.3 \cdot 0.2 = 0.06$$

Part 3

$$P(\text{admitted}) = P(\text{sports injury} \cap \text{admitted}) + P(\text{sports injury}^c \cap \text{admitted}) = 0.3 \cdot 0.2 + 0.7 \cdot 0.4 = 0.34$$

Part 4

$$P(\text{sports injury}|\text{admitted}) = \frac{P(\text{sports injury} \cap \text{admitted})}{P(\text{admitted})} = \frac{0.06}{0.34} = 0.17647 \approx 0.18$$

Question 7

Darren and Charles are friends who often go to the cinema together. On such visits there is a probability of 0.4 that Darren will buy popcorn. The probability that Charles will buy popcorn is 0.7 if Darren buys popcorn and 0.35 if he doesn't.
When Darren and Charles visit the cinema together:

(a) Find the probability that both buy popcorn

(b) Show that the probability that neither buys popcorn is 0.39

(c) Find the probability that exactly one of them buys popcorn

Sarah sometimes join Darren and Charles on their cinema visits. On these occasions, the probability that sarah buys popcorn is 0.55 if both of charles and darren buy popcorn and 0.25 if exactly one of darren and charles buys popcorn
When Darren, Sarah and Charles visit the cinema together:

(a) What is the probability that the three of them buy popcorn

(b) Charles and sarah buy popcorn but Daren Doesn't

My Solution

Given:
$P(\text{Darren buys popcorn}) = 0.4$
$P(\text{Charles buys popcorn}|\text{Darren buys popcorn}) = 0.7$
$P(\text{Charles buys popcorn}|\text{Darren doesn't buy popcorn}) = 0.35$
$P(\text{Darren doesn't buy popcorn}) = 0.6$

Part 1

$$\begin{aligned}
&P(\text{Darren buys popcorn} \cap \text{Charles buys popcorn})\\
&= P(\text{Darren buys popcorn}) \cdot P(\text{Charles buys popcorn}|\text{Darren buys popcorn})\\
&= 0.4 \cdot 0.7\\
&= 0.28
\end{aligned}$$

Part 2

$$\begin{aligned}
&P(\text{Darren doesn't buy popcorn} \cap \text{Charles doesn't buy popcorn})\\
&= P(\text{Darren doesn't buy popcorn}) \cdot P(\text{Charles doesn't buy popcorn}|\text{Darren doesn't buy popcorn})\\
&= 0.6 \cdot 0.65\\
&= 0.39
\end{aligned}$$

Part 3

$$\begin{aligned}
&P(\text{Darren buys popcorn} \cap \text{Charles doesn't buy popcorn or Charles buys popcorn} \cap \text{Darren doesn't buy popcorn})\\
&= P(\text{Darren buys popcorn}) \cdot P(\text{Charles doesn't buy popcorn}|\text{Darren buys popcorn})\\
&+ P(\text{Charles buys popcorn}) \cdot P(\text{Darren doesn't buy popcorn}|\text{Charles buys popcorn})\\
&= 0.4 \cdot 0.3 + 0.6 \cdot 0.35\\
&= 0.33
\end{aligned}$$

Question 7 continued on next page. . .

Part 4

Given: $P(\text{Sarah buys popcorn}|\text{Darren buys popcorn} \cap \text{Charles buys popcorn}) = 0.55$
$P(\text{Sarah buys popcorn}|\text{Darren buys popcorn} \cap \text{Charles doesn't buy popcorn}) = 0.25$
$P(\text{Sarah buys popcorn}|\text{Darren doesn't buy popcorn} \cap \text{Charles buys popcorn}) = 0.25$

$$\begin{aligned} P(\text{3 buy popcorn}) &= P(\text{Darren buys popcorn} \cap \text{Charles buys popcorn} \cap \text{Sarah buys popcorn}) \\ &= 0.4 \cdot 0.7 \cdot 0.55 \\ &= 0.154 \end{aligned}$$

Part 5

$$\begin{aligned} P(\text{Charles buys popcorn} \cap \text{Sarah buys popcorn} \cap \text{Darren doesn't buy popcorn}) &= 0.6 \cdot 0.35 \cdot 0.25 \\ &= 0.0525 \end{aligned}$$

Question 8

Following a flood, 120 tins were recovered from Lee's corner shop. Unfortunately, the water had washed off all the labels. Of the tins, 50 contained pet food, 20 contained peas, 35 contained beans and the rest contained soup.

(a) Lee selects a tin at random, what is the probability that it contains soup?

(b) Lee selects a tin at random, what is the probability that it doesn't contain pet food

(c) Lee selects two tins at random (without replacement), what is the probability that both contains peas

(d) Lee selects two tins at random (without replacement), what is the probability that one contains pet food and the other contains peas

(e) Lee selects 3 tins at random (without replacement). What is the probability that one contains pet food, one contains peas and one contains beans.

(f) Find the probability that Lee will have to open more than two tins before he finds one that doesn't contain pet food.

My Solution

Part 1

Soup left: 120 - 50 - 20 - 35 = 15

$$P(\text{soup}) = \frac{15}{120} = 0.125$$

Part 2

$$P(\text{pet food}^c) = \frac{120-50}{120} = 0.58333$$

Part 3

$$P(\text{peas} \cap \text{peas}) = \frac{20}{120} \cdot \frac{19}{119} = 0.00254$$

Part 4

$$P(\text{pet food} \cap \text{peas}) = \frac{50}{120} \cdot \frac{20}{119} \cdot 2 = 0.140056$$

Part 5

$$P(\text{pet food} \cap \text{peas} \cap \text{beans}) = \frac{50}{120} \cdot \frac{20}{119} \cdot \frac{35}{118} \cdot 3! = 0.12462612164 \approx 0.125$$

Part 6

What are the chances that it's just

$$\frac{50}{120} \cdot \frac{49}{119} = 0.1715686 \approx 0.172$$

Question 9

A school employs 75 teachers. The following table summarizs their length of service at the school, classified by gender.

	<3 years	3 years to 8 years	>8 years
Female	12	20	13
Male	8	15	7

(a) Find the probability that a randomly selected teacher is a female

(b) Find the probability that a randomly selected teacher is female given that the teacher has more than 8 years service

(c) Find the probability that a randomly selected teacher is female given that the teacher has less than 3 years service

(d) State, giving a reason, whether or not the event of selecting a female teacher is independent of the event of selecting a teacher with less than 3 years service

My Solution

Part 1

$$P(\text{female}) = \frac{12+20+13}{75} = \frac{45}{75} = 0.6$$

Part 2

$$P(\text{female}| > 8 \text{ years}) = \frac{P(\text{female}\cap > 8 \text{ years})}{P(> 8 \text{ years})} = \frac{\frac{13}{75}}{\frac{20}{75}} = 0.65$$

Part 3

$$P(\text{female}| < 3 \text{ years}) = \frac{P(\text{female}\cap < 3 \text{ years})}{P(< 3 \text{ years})} = \frac{\frac{12}{75}}{\frac{20}{75}} = 0.6$$

Part 4

Test for independence:

$$P(\text{female}| < 3 \text{ years}) = P(\text{female})$$

Since both are 0.6, the events are independent.

Question 10

A group of students bought, in total, 25 items of clothing at two shops: Mango and Iora. The following table shows how many tops, jeans and sweaters were bought at each of the two shops:

	Top	Jeans	Sweaters
Mango	3	7	5
Iora	2	5	3

One item of clothing is chosen at random from these 25 items:

(a) What is the probability that the chosen item is a top?

(b) What is the probability that the chosen item was bought from Iora?

(c) What is the probability that the chosen item is a top and was bought from Iora?

(d) State with a reason whether the events (chosen item is a top) and (chosen item was bought from iora) are independent.

(e) Given that the chosen item is not a top, find the conditional probability that it was

My Solution

	Top	Jeans	Sweaters	Total
Mango	3	7	5	15
Iora	2	5	3	10
Total	5	12	8	25

Part 1

$$P(\text{top}) = \frac{5}{25} = 0.2$$

Part 2

$$P(\text{Iora}) = \frac{10}{25} = 0.4$$

Part 3

$$P(\text{top} \cap \text{Iora}) = \frac{2}{25} = 0.08$$

Part 4

$$P(\text{top}|\text{Iora}) = \frac{P(\text{top} \cap \text{Iora})}{P(\text{Iora})} = \frac{0.08}{0.4} = 0.2$$

Test for independence:

$$P(\text{top}) = P(\text{top}|\text{Iora})$$

Since both are 0.2, the events are independent.

Part 5

$$P(\text{Iora}|\text{top}^c) = \frac{P(\text{Iora} \cap \text{top}^c)}{P(\text{top}^c)} = \frac{0.4 \cdot 0.8}{0.8} = 0.4$$

Question 11

A volunteer for the Health Literacy Center was investigating the attitudes of students towards smoking on campus. A random sample of 730 students from all four grade levels was taken. Each student was given the statement "Smoking is dangerous for your health" and asked whether they agreed, had no opinion or disagreed. The following contingency table summarizes the results.

(a) Complete the following table knowing that

  (a) 387 Male students participated

  (b) 277 students agreed and 97 students had no opinion

  (c) 154 from those who agreed were female

  (d) Only 36 from the males who participated had no opinion

(b) Among the participants, what is the probability that a student agreed P (A)?

(c) Show whether the events of students agreeing(A) and being a female student (F) are independent

(d) What is the probability that a student is a female given that the student had no opinion $P(F|N)$? What is the probability that a student disagreed, given that the student is Male $P(D|M)$?

(e) Use Bayes' Theorem to calculate the probability $P(M|A)$

My Solution

Part 1

	Agree (A)	No Opinion (N)	Disagree (D)	Totals
Male (M)	123	36	228	387
Female (F)	154	61	128	343
Totals	277	97	356	730

Part 2

$$P(A) = \frac{277}{730} = 0.38$$

Part 3

$$P(A|F) = \frac{P(A \cap F)}{P(F)} = \frac{0.21}{0.47} = 0.45$$

Test for independence:

$$P(A) = P(A|F)$$

Since $P(A) = 0.38$ and $P(A|F) = 0.45$, the events are not independent.

Part 4

$$P(F|N) = \frac{P(F \cap N)}{P(N)} = \frac{\frac{61}{730}}{\frac{97}{730}} = 0.63$$

$$P(D|M) = \frac{P(D \cap M)}{P(M)} = \frac{\frac{228}{730}}{\frac{387}{730}} = 0.59$$

Question 11 continued on next page. . .

Part 5

$$P(M|A) = \frac{P(M \cap A)}{P(A)} = \frac{\frac{123}{730}}{\frac{277}{730}} = 0.44$$

Question 12

Letter A is sent by first-class post and has a probability of 0.9 of being delivered next day. Letter B is sent by second-class post and has a probability of only 0.3 if being delivered the next day.

(a) Find the probability that both letters are delivered the next day

(b) Find the probability that neither letter is delivered the next day

(c) Find the probability that at least one of the letters is delivered the next day

(d) Given that at least one of the letters is delivered the next day, find the probability that letter A is delivered the next day

My Solution

Draw the Probability tree diagram to solve the problem.

Part 1

$$P(A \cap B) = P(A) \cdot P(B) = 0.9 \cdot 0.3 = 0.27$$

Part 2

$$P(A^c \cap B^c) = P(A^c) \cdot P(B^c) = 0.1 \cdot 0.7 = 0.07$$

Part 3

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.9 + 0.3 - 0.27 = 0.93$$

OR, Using a tree diagram:

$$P(A \cup B) = 0.9 \cdot 0.3 + 0.9 \cdot 0.7 + 0.1 \cdot 0.3 = 0.93$$

Part 4

$$P(a|\text{at least 1 letter}) = \frac{P(a \cap \text{at least 1 letter})}{P(\text{at least 1 letter})} = \frac{0.9 \cdot 0.3 + 0.9 \cdot 0.7}{0.93} = 0.9677419 \approx 0.9677$$

Question 13

ANSWERS

1. (b) $\frac{3}{4}$, $c = \frac{1}{2}$, classical interpretation
2. $\frac{7}{15}$, $\frac{77}{150}$, $\frac{8}{15}$, Relative frequency interpretation
3. 0.9
4. 0.882, 0.1694
5. 0.2,0.3, 0.2/0.3
6. 0.3, 0.06, 0.34, 0.17647
7. 0.28, 0.39, 0.33,0.154, 0.0525
8. 0.125, 0.0583, 0.0266, 0.140, 0.125, 0.172
9. 0.6, 0.65, 0.6, independent
10. $\frac{1}{5}$, $\frac{2}{5}$, $\frac{2}{25}$, independent, $\frac{2}{5}$
11. B)P(A) = $\frac{377}{730}$, C) not Independent, D) $\frac{61}{97}$, $\frac{228}{387}$, E) 0.44404
12. A) 0.27, B)0.07, C)0.93, D)0.9677

11a)

	Agree (A)	No Opinion (N)	Disagree (D)	Totals
Male (M)	123	36	228	387
Female (F)	154	61	128	343
Totals	277	97	356	730