

**INF 1004 Mathematics 2**  
**Tutorial #9**

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**Question 1**

Compute  $A^\top A$  for

$$A = \begin{bmatrix} 2 & -3 \\ -6 & -9 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$

**My Solution**

## Question 2

Compute the inverse of

$$A_0 = \begin{bmatrix} -2 & -3 \\ -6 & -4 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

Use these inverses to solve

$$A_0x = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$A_1x = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

Note: it is not common to solve  $Ax = b$  using matrix inversion.

Reasons:

- $Ax = b$  can be solvable when  $A$  is not invertible.
- It is often slower / more costly

## My Solution

### Question 3

Compute the determinant of

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -4 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 2 \\ -1 & 3 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -5 & -10 & -15 \\ 6 & 12 & 18 \end{bmatrix}$$

- Are they Invertible?
- Which of them has full rank? Which of them has lower rank and which one?

**My Solution**

**Question 4**

For what value  $a$  the matrix is not invertible?

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 4 \\ -3 & 1 & 2 \end{bmatrix}$$

**My Solution**

## Question 5

Compute and apply the Householder matrix which makes transforms the first column of to a multiple of the first one-hot vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  for

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

and for (hint: here subtracting is nicer)

$$A = \begin{bmatrix} 1 & -4 & 3 \\ 3 & 1 & 1 \\ \sqrt{6} & 3 & 1 \end{bmatrix}$$

## My Solution

**Question 6**

Verify that

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

Satisfies being an orthogonal matrix.

**My Solution**

**Question 7**

What is the cosine of the angle between

$$(6, -6, -4, \sqrt{12}), (6, 4, 2, \sqrt{25})?$$

**My Solution**



## Question 8

Another  $3 \times 3$  affine system

- show the intermediate result when the first column is the one hot vector  $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$  for the first time
- show that the intermediate result when the matrix has row echelon form for the first time
- get the solution

$$2x - 3y + 2z = -4$$

$$7x + 4.5y - 1z = 16$$

$$4x + 3y + z = 2$$

## My Solution