Linear Algebra L1 - Vectors

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Learning Goals

- vectors of real numbers
- norms of vectors and their properties
- inner products, their interpretation and properties
- representing a vector as a linear combination
- vector spaces
- independent sets of vectors
- orthogonal sets of vectors
- projecting onto a vector, removing the direction of a vector
- creating an orthogonal set of vectors

Task 1

Compute the euclidean vector norm for vectors

$$[1, 0, 2],\ [3, 4],\ [-7, 2, -4, \sqrt{12}]$$

Task 2

Compute the corresponding unit length vector for these:

$$[3, 4],\ [-1, -2, 3],\ [-7, 2, -4, \sqrt{12}]$$

Task 3

Compute the inner product between these vectors and their angle in degrees:

$$[3, -2, 2], [1, 2, 2]$$

Compute the inner product between these vectors and their angle in degrees:

$$[1, 0, 1], [2, 1, -2], [\frac{1}{2\sqrt{2}}, -\frac{\sqrt{3}}{2}, \frac{1}{2\sqrt{2}}]$$

Task 4

- What is the projection of $[5, 2]$ onto the subspace spanned by vector $[1, 1]$?
- What is the projection of $[0, 2, 1]$ onto the subspace spanned by vector $[1, -1, -1]$?
- Project $[5, 2]$ onto the subspace spanned by vectors $[2, 3],\ [1, 1]$
- What is the projection of $[1, -1, 1]$ onto the subspace spanned by vectors $[0, 0, -1],\ [2, 0, 1]$? Hint: this one is more tricky. Reason: $[0, -1, -1] \cdot [2, 0, 1] \neq 0$

Task 5

compute these matrix multiplications:

$$\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \end{bmatrix}$$

Question: Do you need more of them to practice? If so, you can do at home:

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 2.5 \\ -3.5 & 1.5 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

Task 6

- Project $[5, 2]$ onto the orthogonal space of vector $[2, -3]$
- Project $[1, -1, 3]$ onto the orthogonal space of vector $[-3, 1, 1]$
- Project $[1, -1, 3, 1]$ onto the orthogonal space of vectors $[-2, 2, 0, 0]$, $[0, 0, \sqrt{2}, \sqrt{2}]$

Task 7

run Gram-Schmid-orthogonalization on the vectors

$$[12, 12, 6], [2, -2, 4], [-2, -2, 1]$$

Task 8: understanding distances coming from ℓ_p-norms

Coding: plot in python or similar the set of points $x \in \mathbb{R}^2$ such that $\|x\|_p = 1$ for

- $p = 0.2$
- $p = 0.5$
- $p = 1$
- $p = 1.5$
- $p = 2$

- $p = 4$
- $p = 8$
- $p = 16$

Hint: in 2 dimensions for $p = 2$ the solution is given by

$$x(t) = (\cos(t), \sin(t))$$

due to $\cos^2(t) + \sin^2(t)$ =1.
You can use the same idea with different powers. You can start by considering $(\cos^r(t), \sin^r(t))$. One thing to note: $\cos(t)^r, \sin(t)^r$ is not always defined for negative values and certain r.

For $p \neq 2$ you can consider this, which deals with the signs:

$$x(t) = (sign(\cos(t))|\cos(t)|^r, sign(\sin(t))|\sin(t)|^r)$$

for the right choice of r. Find out which r is suitable for a general $p > 0$ such that $\|x\|_p = 1$. Then plot it in python.