INF 1004 Mathematics 2
Tutorial #6

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Question 1

A sample of people have been assigned to fly to Mars. The frequencies of those in each age-group are given in the table below. Frequency represents the total number of people in each age range. Note, all ages are only measured (discretely) in years, and each range is inclusive of its boundaries.

a) What is the Mode range for these data?

b) What is the interpolated Median (Q) for these data?

c) Calculate the Mean for these data.

Age Range	Frequency
1 to 8	2
9 to 20	14
21 to 40	16
41 to 60	93
61 to 80	140
81 to 105	35

Sample Solutions

Age Range	Frequency	Midpoint	cf	xf
1 to 8	2	4.5	2	9
9 to 20	14	14.5	16	203
21 to 40	16	30.5	32	488
41 to 60	93	50.5	125	4969.5
61 to 80	140	70.5	265	9870
81 to 105	35	93	300	3255
Total	300			18521.5

Part 1

Modal Range: 61 to 80

Part 2

$$
\begin{aligned}
Q &= L_m + \left(\frac{\frac{n}{2} - cf_{m-1}}{f_m}\right) w_m \\
&= 61 + \left(\frac{\frac{300}{2} - 125}{140}\right)(80 - 61 + 1) \\
&= 64.5714 \\
&\approx \boxed{64.57}
\end{aligned}
$$

Note: If you used $(80 - 61)$, Even though this is incorrect for this inclusive, discrete range, you were still awarded **most of the marks**. `Ans:` $Q = 64.3929$

Part 3

$$
\begin{aligned}
\bar{x} &= \frac{\sum x f_i}{n} \\
&= \frac{(4.5)(2) + (14.5)(14) + (30.5)(16) + (50.5)(93) + (70.5)(140) + (93)(35)}{300} \\
&= \frac{18521.5}{300} \\
&= 61.7383 \\
&\approx \boxed{61.74}
\end{aligned}
$$

Question 2

You are taking random samples of seashells on a beach. The seashells are either univalves or bivalves. You know that 76% of seashells on the beach are univalves. You pick 100 random samples of seashells from the beach. Taking each seashell is independent from the others and the probability of selecting a univalve or a bivalve remains the same for each sample.

a) Given that you hope to pick as many bivalves as possible, answer the following.

b) What are the mean (expected value) and variance for taking 100 random samples of seashells from the beach?

c) What is the probability that you select exactly one bivalve seashell in your 100 samples?

d) What is the probability of collecting more than <u>86%</u> univalve seashells in the 100 samples?

Sample Solutions

$$X \sim B(100, 0.24)$$

How to Identify:

1. Finite number of identical trials (for example, each coin flip, etc)
2. Each trial must have one of two discrete outcomes (success or failure)
3. Probability of success and failure is fixed over all trials
4. Fixed number of independent trials

Part 1

$$\begin{aligned} E(x) &= np \\ &= 100 \cdot 0.24 \\ &= \boxed{24} \end{aligned}$$

$$\begin{aligned} Var(x) &= np(1-p) \\ &= 100 \cdot 0.24 \cdot 0.76 \\ &= \boxed{18.24} \end{aligned}$$

Part 2

$$\begin{aligned} P(X=1) &= \binom{100}{1}(0.24)^1(0.76)^{100-1} \\ &= 3.8085 \times 10^{-11} \\ &\approx \boxed{3.81 \times 10^{-11}} \end{aligned}$$

Question 2 continued on next page...

Part 3

Solution 1:

$$X \sim B(100, 0.24)$$

more than 86 univalves $\rightarrow 87, 88 \ldots 100$ or $13, 12 \ldots 0$ bivalves

$$np = 24 \quad nq = 76$$

Both > 5, hence can approximate to standard normal

$$\sigma^2 = 100(0.24)(0.76) = 18.24$$
$$X \sim N(24, 18.24)$$

$$\begin{aligned}
P(X \leq 13) &= P(X < 13.5) \\
&= P(Z < \frac{13.5 - 24}{\sqrt{18.24}}) \\
&= P(Z < -2.4585) \\
&= P(Z < -2.46) \\
&= 1 - 0.9931 \\
&= \boxed{0.0069}
\end{aligned}$$

Note: Also Accepting full binomial probability summation $P(X = 0) + \ldots + P(X = 13) = 0.0047$

Solution 2: (My Solution)

Let Y be the number of univalves

$$Y \sim B(100, 0.76)$$
$$np = 76 \quad nq = 24$$

Both > 5, hence can approximate to standard normal

$$\sigma^2 = 100(0.76)(0.24) = 18.24$$

$$\begin{aligned}
P(X > 86) &= P(Y > 86.5) \\
&= P(Z > \frac{86.5 - (100 \cdot 0.76)}{\sqrt{18.24}}) \\
&= P(Z > 2.4585) \\
&= P(Z > 2.46) \\
&= 1 - P(Z \leq 2.46) \\
&= 1 - 0.9931 \\
&= \boxed{0.0069}
\end{aligned}$$

Notes:
- If you forgot the 0.5 approximation, you get 2.3415, $P(Z > 2.34) = 1 - 0.9904 = 0.0096$(sliightly worse)
- If you use $z = 2.45$, you will get a slightly worse probability $= 1 - 0.9929 = 0.0071$
- Also Accepting the full binomial probability summation $P(X = 87) + \ldots + P(X = 100) = 0.0047$

Question 3

You have landed on an alien planet. When you go out to explore the planet you are guaranteed to encounter one alien. There are three types of aliens: Circles, Squares, and Triangles. You know that the chance of encountering a Circle is 50%, Square is 30% and Triangle is 20%.

Unfortunately, the aliens are sometimes unfriendly and may attack. This likelihood of being attacked depends on the type of alien you encounter. If you encounter (given) a Circle it has a 10% chance of attacking you, encountering (given) a Square has a 30% chance of attacking you, while encountering (given) a Triangle has an 80% chance of attacking you.

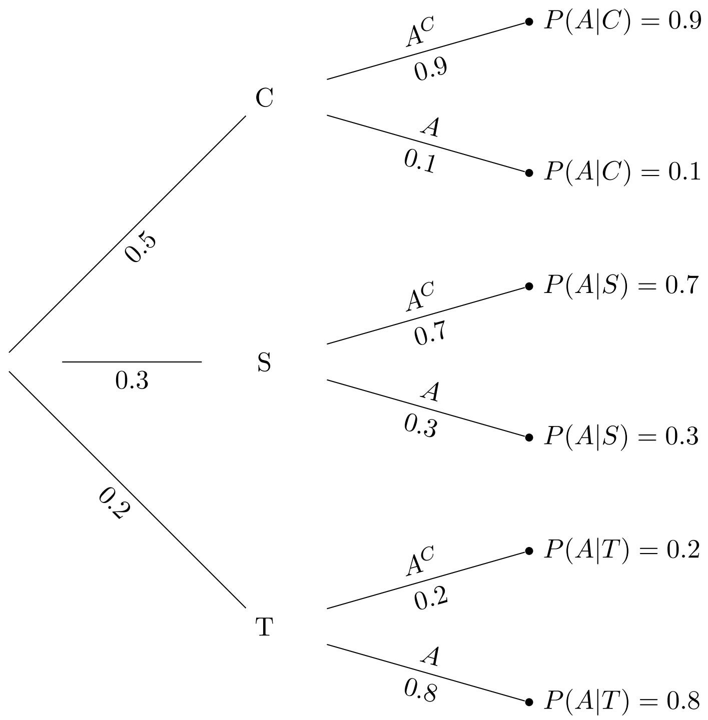
Given this information, answer the following questions.

a) When you explore the planet, what is the probability of you being attacked by an alien?

b) When you explored the planet: (given) if you were NOT attacked by an alien, what is the probability that you encountered a Circle?

c) When you explored the planet: (given) if you were attacked by an alien, what is the probability that you encountered a Triangle?

Sample Solutions



$P(C)$	0.5
$P(S)$	0.3
$P(T)$	0.2
$P(A\|C)$	0.1
$P(A\|S)$	0.3
$P(A\|T)$	0.8

Question 3 continued on next page. . .

Part 1

$$
\begin{aligned}
P(a) &= P(C)P(A|C) + P(S)P(A|S) + P(T)P(A|T) \\
&= (0.5)(0.1) + (0.3)(0.3) + (0.2)(0.8) \\
&= \boxed{0.3})
\end{aligned}
$$

Part 2

$$
\begin{aligned}
P(C|A^C) &= \frac{P(C \cap A^C)}{P(A^C)} \\
&= \frac{(0.5)(0.9)}{(0.5)(0.9) + (0.3)(0.7) + (0.2)(0.2)} \\
&= 0.6428571429 \\
&\approx \boxed{0.6429}
\end{aligned}
$$

Part 3

$$
\begin{aligned}
P(T|A) &= \frac{P(T \cap A)}{P(A)} \\
&= \frac{0.2 \cdot 0.8}{0.3} \\
&= 0.5333333333 \\
&\approx \boxed{0.5333}
\end{aligned}
$$

Question 4

Flying from earth to a distant planet may take a variety of different times, depending on random factors of space flight. The times are normally distributed with a mean time of 40 weeks and standard deviation of 5 weeks.

Given this information, answer the following.

a) What is the probability that flying from earth to the planet takes more than 35 weeks?

b) What is the probability that flying from earth to the planet takes less than 30 weeks?

c) What is the probability that flying from earth to the planet takes between 25 and 45 weeks?

Sample Solutions

$$X \sim N(40, 5^2)$$

Part 1

$$\begin{aligned} P(X > 35) &= P(Z > \frac{35-40}{5}) \\ &= P(Z > -1) \\ &= P(Z \leq 1) \\ &= \boxed{0.8413} \end{aligned}$$

Part 2

$$\begin{aligned} P(X < 30) &= P(Z < \frac{30-40}{5}) \\ &= P(Z < -2) \\ &= P(Z \geq -2) \\ &= 1 - P(Z < -2) \\ &= 1 - 0.9772 \\ &= \boxed{0.0228} \end{aligned}$$

Part 3

$$\begin{aligned} P(25 < X < 45) &= P(25 < Z < 45) \\ &= P(\frac{25-40}{5} < Z < \frac{45-40}{5}) \\ &= P(-3 < Z < 1) \\ &= P(Z \leq 1) - P(Z \leq -3) \\ &= P(Z \leq 1) - (1 - P(Z \leq 3)) \\ &= 0.8413 - (1 - 0.9987) \\ &= \boxed{0.84} \end{aligned}$$

Question 5

Revision

What we have covered

1. Descriptive Statistics - types of data, mean/median/mode, standard deviation, variance, IQR, range, CoF
2. Probability Theorem - independent, mutually exclusive, Baye's
3. Discrete Probability Distributions $X \sim B(n, p)$
4. Continuous Probability Distributions $X \sim N(\mu, \sigma^2)$
5. Binomial Approximation Using Normal, with continuity correction
6. Sampling Distributions - Process of stats studies, sampling methods.

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$